

Strategic regulation of a multi-national banking industry

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Abstract

This paper focuses on the consequences of cross-border banking and entry of multi-national bank (MNB) subsidiaries for banking supervision and regulation. When a MNB expands internationally with subsidiaries, the MNB operates under the legislation of several countries - both the home country and the host countries. Although these countries have agreed upon minimum standards and supervisory principles, such as in the EU directives or the Basle Accords, substantial degrees of freedom are still left to the national regulators. An interesting and important issue is whether the decentralized approach to regulation of MNBs creates inefficiencies and financial instability. Host country regulation of MNB subsidiaries creates cross-border externalities, where the supervisors and regulators in one country will be concerned with the standards in the home country and in other host countries. Our main result is that lack of international coordination of banking regulation works to lower capital adequacy requirements. In equilibrium, however, regulators respond by increasing the incentives to improve asset quality, making the probability of banking failure insensitive to the decentralized nature of banking regulation. Ownership of the MNB is shown to be of importance for the outcome of strategic banking regulation.

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1 Introduction

The banking industry is becoming more international. Technological changes allow financial markets to integrate and regulatory changes have lowered barriers for cross-border banking. By 1996, total assets of overseas branches and subsidiaries of US banks exceeded \$1.1 trillion. In the same year, 58 per cent of UK loans were made by branches and subsidiaries of non-UK banks. In Germany, 17 per cent of private commercial bank loans were made by non-German banks.¹ In spite of globalization, the banking industry is still one of the most regulated industries in the world (Santos (2000)). Banks in most countries have to meet solvency standards and reserve requirements, pay deposit insurance premiums, and accept various forms of monitoring of their risk management systems and of their individual transactions (ensuring, for instance, that adequate collateral was put up), etc.. The combination of extensive regulation and a trend towards integration of financial markets raises new issues with respect to international harmonization of bank regulation.

Cross-border banking may take several forms. A bank holding company may expand business internationally by lending directly to customers abroad from its domestic offices. Other possibilities is to set up branches or subsidiaries abroad which may raise deposits and grant loans. With respect to regulation there is an important distinction between branches and subsidiaries.

Branches established abroad are legally an integrated part of the parent bank, and, therefore is under the regulation of the home country. The European Union's (EU's) single market and principle of "one single licence" allow parents banks with a licence from an EU-country to set up branches anywhere within EU. The parent bank needs to respect the regulatory framework of the home country. *Subsidiaries* are separated as legal entities from the parent bank. These entities are separately capitalized, and may therefore fail independently. Consequently, the subsidiary needs its own banking licence, and must respect the regulatory framework of the host country. Although subsidiaries are treated as a separate bank by the host country, all subsidiaries are owned (at least with majority) by the parent bank. As the owner, the bank holding company is able to control important decisions with respect to business strategy. Resources and skills within the holding company may thus be transferred to its subsidiaries.

EU figures show that subsidiaries are important part in the ongoing integration (ECB,1999). In Ireland, the market share of foreign subsidiaries was 35 per cent in 1997. In the UK, Portugal, Spain and the Netherlands the market share of foreign subsidiaries inn 1997 was in the range of 8 to 5 per cent. Other markets such as Italy, Greece and Denmark have experience less entry from foreign subsidiaries. In the Nordic countries, the largest bank

¹Figures provided by Calzolari and Loranth (2001).

Nordea has chosen to compete in the Nordic market with subsidiaries in each of the countries Finland, Sweden, Norway and Denmark.

This paper focuses on the consequences of multi-national bank (MNB) subsidiaries for banking supervision and regulation. When a MNB expands internationally with subsidiaries, the MNB will operate under the legislation of several countries - both the home country and the host countries. Although these countries have agreed upon minimum standards and supervisory principles, such as in the EU directives or the Basle Accords, substantial degrees of freedom are still left to the national regulators. An interesting and important issue is whether the decentralized approach to regulation of MNBs creates inefficiencies and financial instability. Host country regulation of MNB subsidiaries creates cross-border externalities, where the supervisors and regulators in one country will be concerned with the standards in the home country and in other host countries.

As noted by Rochet (1999), there is a trend towards more flexible approaches for regulating banks that take into account the decentralized information of individual banks. He states that "this means that the adverse selection paradigm of contract theory is relevant for studying banking regulation". Giammarino et al. (1993) is an early example of using a contract theory approach to the study of optimal banking regulation. We adopt their framework to study strategic (non-cooperative) regulation of MNBs.

Strategic regulation is modelled as multi-principal regulation of a MNB that allocates resources towards activities that increase asset quality. Our main result is that lack of international coordination of banking regulation works to lower capital adequacy requirements. In equilibrium, however, regulators respond by increasing the MNB's incentives to improve asset quality, making the probability of banking failure insensitive to the decentralized nature of banking regulation. Ownership of the MNB is shown to be of importance for the outcome of strategic banking regulation. If the MNB is owned by shareholders from outside the market operated by the bank ("third-country shareholders"), the regulatory regime becomes more distortive since the regulators then become more eager to extract banking profit. With more "inside-shareholders", the regulatory policy becomes more pro-bank industry oriented. Therefore, with a trend towards more flexible approaches to banking regulation, we would expect "third-country"-owned MNBs to be handicapped in the market.

Although international coordination of regulation and supervision, and the issue of "level playing fields" in financial markets have been high on the political agenda, theoretical studies have until recently been rare. Calzolari and Loranth (2001) survey specific regulatory issues brought about by MNBs with particular attention to solvency and prudential regulation. Calzolari and Loranth (2002) develops a model to analyze the incentives of home and host country regulators to intervene with prudential actions in MNBs. Both branch- and subsidiary-organized MNBs are considered. The policy

decision of the regulators is whether or not to close the bank, based on the received information about the quality of the bank. When there is complete information exchange between the regulators, they show that the host country regulator of a subsidiary-organized MNB has less incentive to intervene than the home country regulator of a branch-organized MNB.

Dell’Ariccia and Marquez (2001) analyze the incentives for independent domestic bank regulators to coordinate regulatory policy. Their model is consistent with branch-organized MNBs that makes banks working under different regulatory regimes meet in the same market. After identifying a so-called “race to the bottom” without coordination, they investigate the conditions under which regulators would benefit from coordination and, hence, giving up independence. Somewhat relatedly, Sinn (2001) shows that there will be undersupply of regulation due to what he calls systems competition. A positive externality of the national solvency regulation explains the undersupply of such regulation. Boot, Dezelan and Milbourn (2000), investigate the importance of a level playing field in a simple industrial organization model of banking competition. The cost of regulation, in terms of lost profit, is larger when the regulated banks compete in a market with other non-regulated banks. A recent paper by Stolz (2002) introduces interbank market in a model similar to Giammarino et al. (1993). Assuming a subsidiary-organized MNB, as in the present paper, she shows that a national supervisor/regulator will not adequately internalise costs imposed on other economies by hazardous banking behaviour in her jurisdiction. The cross-country contagion effect caused by interbank lending will not be internalized by supervisors with a national mandate only.

The present paper differs from the above in several aspects. We formulate the regulatory game under decentralization as a common-agency. This allows us to derive the national regulator’s optimal response to MNBs, and, further, to identify the sources of regulatory inefficiency. As pointed out above, our analysis provides insights into the importance of bank ownership under decentralized regulation.

The rest of paper is structured as follows. Section 2 describes the model. Section 3 derives the optimal regulatory policy under international coordination. Section 4 derives the regulatory equilibrium without coordination. In section 5 the importance of ownership is discussed. Section 6 derives explicit solutions for regulatory policies by assuming specific functional forms. Section 7 concludes.

2 The Model

We consider a multinational bank (MNB) with subsidiaries in two different countries, $i = 1, 2$. Each subsidiary is operating under the legislation of the host country. In each of the two countries there are a number of firms

having access to risky investment projects that need external funding, and bank loans are assumed to be their only source of funding. The average return of the investment projects in each country is treated as a random variable with a distribution $G(r|q)$ defined over $[\underline{r}, \bar{r}]$. Here q is the quality of the loan portfolio acquired by the bank. An increase in quality shifts the distribution of return in the sense of first-order stochastic dominance, i.e. $G_q(r|q) \leq 0 \ \forall \ r \in [\underline{r}, \bar{r}]$. The two countries are assumed to be identical as far as business environments are concerned. Given the quality of the loan portfolio, therefore, the distribution of average return is the same in both countries. In both countries there is also a risk-free asset with a rate of return equal to 1.

Loan quality. The quality of the loan portfolio in the two subsidiaries is assumed to be a function of the innate quality of the investment projects, the amount of resources devoted to auditing and screening of the investment projects, and country-specific macroeconomic conditions. Following our assumption of identical business environments, the innate quality of the projects is assumed to be identical, and denoted by θ . The amount of resources devoted to auditing in a subsidiary is given by e_i . Adding local macroeconomic conditions β_i , the quality of the loan portfolio in country i is assumed to be given by

$$q_i = \beta_i \theta + e_i. \quad (1)$$

The regulator is able to observe the realized quality of the loan portfolio in its jurisdiction. This assumption is consistent with the periodic inspections of bank assets that regulators undertake in practice. Admittedly, inspections and supervisions provide only imperfect measures of asset quality, but as a simplifying assumption this is justified by the fact that these (imperfect) measures are valuable and considered to be important by the regulator. Although the final asset quality is observed by the regulator, the MNB has private information about the innate quality of the investment projects θ . Having observed a given asset quality, the regulator does not know the amount of resources the bank needed to spend on auditing in order to achieve this level.

It is common knowledge that innate quality is distributed according to a cumulative distribution function $F(\theta)$, with density $f(\theta)$ over an interval $[\underline{\theta}, \bar{\theta}]$. The bank and the regulator are assumed to know the macroeconomic situation. If there is a recession $\beta_i = \underline{\beta}$. Else $\beta_i = \bar{\beta}$, where $\bar{\beta} > \underline{\beta}$. Later, β will denote $\beta_1 + \beta_2$.

Funding. At the outset, the MNB's only asset is that it has access to the market of risky investment projects in the two countries. Provided that the MNB complies with rules set up by the regulator, the bank can issue deposits and grant loans to firms with investment projects. There are two funding sources available for the bank - outside equity and deposits. The MNB must promise the new shareholders an expected return equal to $r^e E_i$

in order to attract outside equity of size E_i to the subsidiary in country i . r^e (> 1) is an exogenous expected rate of return that makes investors willing to provide equity. The other source of funding is insured deposits D_i . To simply exposition, the bank is assumed to attract deposits of fixed size in both countries. The amount of deposits are normalized to 1, i.e. $D_1 = D_2 = 1$. Deposits are paid an interest rate equal to 1.

The MNB's costs of improving asset quality beyond the base levels ($\beta_1\theta$) and ($\beta_2\theta$) are given by $\psi(e_1 + e_2)$. These costs reduce the MNB's initial wealth. $\psi(\cdot)$ is an increasing and strictly convex function, which implies that the MNB's effort in the two jurisdictions are substitutes, i.e. $\frac{\partial^2 \psi}{\partial e_1 \partial e_2} = \psi'' > 0$. Finally, each subsidiary must satisfy the cash flow constraint

$$L_i + R_i + P_i = D_i + E_i, \quad (2)$$

where L_i is the amount of risky loans granted by the bank in country i , R_i is the amount of risk-free assets kept by the bank, and P_i is the deposit insurance premium paid in order to be allowed to run the bank in jurisdiction i . Following Giammarino et al. (1993), we assume that $D_i = L_i$. Hence, the size of the bank's activity in the two countries, in terms of risky loans, is exogenous. This assumption highlights the important role of equity as a means for adjusting the probability of bank default. Our focus is on the role of regulation and supervision in affecting loan quality and the probability of banking failure.

Expected profit. The expected global profit of the MNB may now be written:

$$\pi = \sum_{i=1}^2 \int_{r_i^b}^{\bar{r}} [r_i + R_i - 1] dG(r_i | q_i) - r^e \sum_{i=1}^2 (R_i + P_i) - \psi\left(\sum_{i=1}^2 q_i - \beta\theta\right) \quad (3)$$

The first term is the expected value of the cash flow earned in the two jurisdictions. r_i^b is the break-even return level. If the average return of a subsidiary drops below this level, the subsidiary fails, and the governmental deposit insurance fund pays the depositors. Note that the break-even level of return depends on the amount of reserves kept by the bank; it is given by $r_i^b = 1 - R_i$. Since equity is used to keep reserves (and to pay the insurance premium), the role of equity is to adjust the probability of default.

Subtracting the second term in (3), which is the cost of funding risk free assets and the insurance premium with outside equity, we get the insider's share of the cash flow. Note that the outside shareholders are not guaranteed a return equal to r^e . Instead, in order to provide capital equal to E_i , they must be given an equity ratio z_i , such that $r^e E_i = z_i \int_{r_i^b}^{\bar{r}} [r_i + R_i - 1] dG(r_i | q_i)$. Outside equity is costly for the insider because its share of the cash-flow $(1 - z_i)$ is reduced. The last term in (3) is the cost of improving loan quality.

Regulator's objective. The objective of the regulator in each jurisdiction is to provide deposit insurance at lowest costs for the society. The net payoff to the government from providing deposit insurance is given by (note that the second term is negative due to the definition of r_i^b)

$$W_i = r^e P_i + (1 + b) \int_0^{r_i^b} [r_i + R_i - 1] dG(r_i \setminus q_i) \quad (4)$$

The first term in the bracket is the value of the insurance premium collected by the regulator, and the last term is the expected loss from a banking failure. b captures the additional bankruptcy cost due to negative externalities. Using (3) to substitute for $r^e P_i$ in (4), we get

$$W_i = \left\{ \begin{array}{l} \sum_{i=1}^2 \{ \int_{r_i^b}^{\bar{r}} [r + R_i - 1] dG(r \setminus q_i) \} + (1 + b) \int_0^{r_i^b} [r + R_i - 1] dG(r \setminus q_i) \\ - r^e \left[\sum_{i=1}^2 R_i + P_j \right] - \psi(\sum_{i=1}^2 q_i - \beta\theta) - \pi \end{array} \right\} \quad (5)$$

The reason why increased bank profit is costly for the regulator, is that the insurance premium collected by the regulator has to be lowered. Noting that global surplus for the regulators W is given by $W_1 + W_2$, W may be written

$$W = \left\{ \begin{array}{l} \sum_{i=1}^2 \{ \int_{r_i^b}^{\bar{r}} [r_i + R_i - 1] dG(r_i \setminus q_i) + (1 + b) \int_0^{r_i^b} [r_i + R_i - 1] dG(r_i \setminus q_i) \} \\ - r^e \sum_{i=1}^2 R_i - \psi(\sum_{i=1}^2 q_i - \beta\theta) - \pi \end{array} \right\} \quad (6)$$

If the regulator's objective function includes domestic bank profit, we will have $W_i = (1 + \lambda)S_i + \delta_i\pi$, where S_i is now the expected payoff to the goverment from the deposit insurance. Here λ is the general equilibrium shadow costs of public funds (assumed equal in the two countries), and δ_i is the ownership share of country i . In this case a regulatory scheme that generates bank profit is less costly for the regulator. The importance of bank ownership is treated in section 5.

3 Cooperative regulation of the MNB

In this section a single regulator maximizes the global surplus of the deposit insurance scheme. The policy instruments available to the regulator are the required amount of risk free asset to be held by the bank R_i and the insurance premium P_i . In effect, this also determines the level of outside equity E_i . In addition, the regulator specifies the level of asset quality the bank should achieve q_i . However, when choosing a regulatory policy, the regulator suffers from asymmetry of information. The regulator does not know the quality of the business environment, which is crucial for the bank's cost of acquiring a certain quality level of the loan portfolio.

Following standard procedures, the regulatory policy can be analyzed in terms of a direct revelation mechanism. In our case, this means that the bank makes a report on the intrinsic quality of the business environment $\hat{\theta}$, and the regulator responds by offering a regulatory package $\{P_i(\hat{\theta}), R_i(\hat{\theta}), q_i(\hat{\theta})\}$, $i = 1, 2$, from a pre-announced menu. According to the *Revelation Principle*, any indirect mechanism that links the reserve requirements and the insurance premium to the asset quality q_i , has its equivalence in a direct mechanism which makes the MNB report its true type θ .

The MNB's profit as a function of reported type $\hat{\theta}$ and the true type θ is given by

$$\pi(\hat{\theta} \setminus \theta) = \sum_{i=1}^2 \left\{ \int_{r_i^b}^{\bar{r}} [r + \hat{R}_i - 1] dG(r_i \setminus \hat{q}_i) - r^e (\hat{R}_i + \hat{P}_i) \right\} - \psi \left(\sum_{i=1}^2 \hat{q}_i - \beta \theta \right) \quad (7)$$

The incentive compatibility constraint (i.e. the truth-telling constraint) is given by

$$\pi'(\theta) = \beta \psi' \left(\sum_{i=1}^2 q_i(\theta) - \beta \theta \right), \quad (8)$$

where $\pi'(\theta) = \frac{\partial \pi(\hat{\theta} \setminus \theta)}{\partial \theta}$ for $\hat{\theta} = \theta$. Integration by parts gives the following expression for expected profits

$$\int_{\underline{\theta}}^{\bar{\theta}} \pi(\theta) dF(\theta) = \beta \int_{\underline{\theta}}^{\bar{\theta}} \psi' \left(\sum_{i=1}^2 q_i(\theta) - \beta \theta \right) (1 - F(\theta)) d\theta \quad (9)$$

where $\pi(\underline{\theta}) = 0$ due to costly rents.

Maximizing expected W w.r.t $\{R_i(\hat{\theta}), q_i(\hat{\theta})\}$ subject to (9) defines the regulatory policy under coordination:

$$\begin{aligned} & \left[\int_{r_i^b}^{\bar{r}} [r_i + R_i - 1] dG_{q_i}(r_i \setminus q_i) + (1+b) \int_0^{r_i^b} [r_i + R_i - 1] dG_{q_i}(r_i \setminus q_i) \right] \\ &= \psi' \left(\sum_{i=1}^2 q_i - \beta \theta \right) + \beta \psi'' \left(\sum_{i=1}^2 q_i - \beta \theta \right) \frac{1 - F(\theta)}{f(\theta)}, \quad i = 1, 2 \end{aligned} \quad (10)$$

$$(1+b)G(r_i^b \setminus q_i(\theta)) + [1 - G(r_i^b \setminus q_i(\theta))] - r^e = 0, \quad i = 1, 2 \quad (11)$$

Increasing the quality of the loan portfolio, the expected cash-flow increases, and the probability of paying the bankruptcy cost b decreases. The optimal policy balances this effect against the auditing costs associated with increased quality (the first term on the right-hand side of (10)) and the information rent captured by the bank (the second term on the right-hand side of (10)). When quality increases the bank's gain from misrepresenting the innate quality θ increases, and this will materialize as increased bank

profit due to the truth-telling constraint. Therefore, private information introduces a distortion in the choice of quality of the loan portfolio in order to improve the regulator's extraction of rents. The distortion entails a reduction in loan quality (less monitoring effort) for all types of banks except the one with the most promising business environment ($\bar{\theta}$).

Condition (11) determines the optimal level of reserve requirements (or outside equity). The cost of outside equity should be balanced against the benefit from reduced bankruptcy costs. As already pointed out by Giannarino et al. (1993), this rule implies that the probability of a banking failure is independent of induced quality of the loan portfolio. A low quality bank, therefore, is induced to hold more equity and to keep more reserves as a buffer against losses. Moreover, private information in banking (inducing lower quality of the loan portfolio), is compensated for by increased reserve requirements. The capital to loan ratio, therefore, should increase as a response to private information.

We may further note that there is a regulatory induced contagion of macroeconomic shocks between the two countries. If one of the two countries experiences an economic downturn ($\beta_i = \underline{\beta}$), the regulatory induced quality of the loan portfolios in both countries are affected with equal strength.² There are two effects at work here. First, the marginal cost associated with a certain level of quality increases (as seen from the first term on the right-hand side of (10)). This works to reduce induced quality in both countries. Second, as seen from the second term on the right-hand side of (10), a low β_i makes rent extraction less important. This works to increase quality. The second effect, however, disappears as the intrinsic quality of the bank loans approaches $\bar{\theta}$.³ Banks of sufficiently high intrinsic quality, therefore, will experience a deterioration of induced quality of the loan portfolio in *both markets* if one of the markets experiences an economic downturn. Banks of sufficiently low intrinsic quality may actually experience an improvement of loan quality in both markets if there is an economic downturn in one of the markets (see the parametric specification in section 6).

Moreover, if one of the two countries experiences an economic downturn and the subsidiary in the other country is induced to reduce the quality of the loan portfolio, the MNB will face higher capital requirements in both markets (in order to keep the probability of a banking failure constant).

We can summarize the findings so far in the following result:

Proposition 1 *(i) Compared with the first best (symmetric information) solution, too little effort is devoted to loan quality improvement when there is*

²The result that loan qualities are affected exactly equally in both countries is in part a consequence of the modelling assumption that the quality variables enter additively in the cost function. Other formulations with qualities being substitutes for the MNB would yield similar, but not equal contagion effects.

³This is due to the "no distortion at the top"-property. The second term disappears when $\theta = \bar{\theta}$ since $F(\bar{\theta}) = 1$.

international coordination of banking regulation; $q_C(\theta) < q_{FB}(\theta)$ for $\theta < \bar{\theta}$.
(ii) Reserve requirements increase as a response to private information. For the parametric specification given in section 6 it is further true that: (iii) an economic downturn in one country causes a deterioration of loan quality in both countries for banks with sufficiently high intrinsic quality ($\theta \geq E\theta$), and an improvement of loan quality in both countries for banks with sufficiently low intrinsic quality ($\theta < E\theta$). (iv) An economic downturn in one country causes an increase in the capital ratio in both countries for banks with sufficiently high intrinsic quality ($\theta \geq E\theta$), and a reduction in the capital ratio in both countries for banks with sufficiently low intrinsic quality ($\theta < E\theta$).

4 Non-cooperative regulation of the MNB

We now turn to a situation in which the two regulators do not coordinate their regulatory policies towards the banking sector. Instead, the regulatory authorities in the two countries choose reserve requirements, insurance premia (i.e. set capital requirements) and set targets with respect to the qualities of the loan portfolios independently. The MNB relates to each regulator separately. They cannot credibly share information and they act non-cooperatively.⁴

We characterize the regulatory policy of country 1 (Country 2 has a analogous problem). The regulator seeks to maximize the expected domestic surplus, subject to incentive compatibility and participation constraints. The regulator of country 1 now has to take into account that its choice of regulatory rules (reserve requirements and insurance premium) has strategic implications for the behavior of country 2.

To take care of the strategic interaction between regulators, the regulatory policy of country 2 is characterized by a policy rule $R_2(q_2)$ and $P_2(q_2)$, specifying the reserve requirements and the premium to be paid in country 2 as a function of the realized loan quality level in that country. (Under relatively mild conditions—essentially unrestricted communication between the agent and each principal—there is no loss of generality in assuming that each country offers such a policy, see Martimort and Stole (2002).) Given the policy of country 2, country 1 chooses its best policy towards the domestic subsidiary. With a slight abuse of notation, let $\tilde{\pi}(R_1, P_1, q_1, \theta)$ denote the MNB's indirect profit function vis-a-vis country 1; i.e.

$$\tilde{\pi}_1(R_1, P_1, q_1, \theta) = \max_{q_2} \pi(R_1, P_1, q_1, R_2(q_2), P_2(q_2), q_2, \theta) \quad (12)$$

⁴There is an established literature on the regulation of multinational enterprises that focuses on tax policy issues. See e.g. Bond and Gresik (1996), Calzolari (2001), Olsen and Osmundsen (2001,2002).

where $\pi(R_1, P_1, q_1, R_2, P_2, q_2, \theta)$ is the MNB's direct profit defined by (3). Let $\tilde{q}_2(q_1, \theta)$ be the MNB's optimal choice in (12); it is given by the first-order condition

$$0 = R'(\tilde{q}_2)[1 - G(1 - R_2(\tilde{q}_2) \setminus \tilde{q}_2)] + \int_{1-R_2(\tilde{q}_2)}^{\bar{r}} [r_2 + R_2(\tilde{q}_2) - 1] dG(r_2 \setminus \tilde{q}_2) - r^e[(R'_2(\tilde{q}_2) + P'_2(\tilde{q}_2))] - \psi'(q_1 + \tilde{q}_2 - \beta\theta) \quad (13)$$

It is important to note that policy measures taken by the regulator in country 1 to influence the domestic quality level q_1 , will induce a response by the firm so that the foreign quality level $q_2 = \tilde{q}_2$ will be affected as well. For a given regulatory policy from the foreign country, the marginal effect $\frac{\partial \tilde{q}_2}{\partial q_1}$ can in principle be found from (13).

Given the policy of country 2, the optimal policy of country 1 can be found by applying the Revelation Principle in the usual way, taking into account that the relevant profit function for the MNB is now the indirect profit function $\tilde{\pi}_1()$ defined by (12).

Incentive compatibility requires that the firm's rent $\pi_1(\theta)$ now satisfies $\pi'_1(\theta) = \frac{\partial \tilde{\pi}_1}{\partial \theta}$. Since we have $\frac{\partial \tilde{\pi}_1}{\partial \theta} = \frac{\partial \pi}{\partial \theta}$ by the envelope property, we see that equations corresponding to (8) and (9) must hold for the rent $\pi_1(\theta)$, with now $q_2 = \tilde{q}_2(q_1, \theta)$ substituted on the RHS of the equations. Specifically, a bank with innate quality $\theta + d\theta$ can always mimic a bank with lower innate quality θ and by that save 'effort' costs amounting to $\beta\psi'd\theta$, so the regulatory scheme in country 1 must allow for this rent differential; i.e. we must have

$$\pi'_1(\theta) = \beta\psi'(q_1(\theta) + \tilde{q}_2(q_1(\theta), \theta) - \beta\theta) \quad (14)$$

Maximization of the expected value of the national objective W_1 given in (4), subject to IC constraints represented by the equivalent of (9), and taking account of (13), then leads to the following first-order conditions

$$\begin{aligned} & \left[\int_{r_1^b}^{\bar{r}} [r_1 + R_1 - 1] dG_{q_1}(r_1 \setminus q_1) + (1+b) \int_0^{r_1^b} [r_1 + R_1 - 1] dG_{q_1}(r_1 \setminus q_1) \right] \\ &= \psi'(q_1 + \tilde{q}_2 - \beta\theta) + \beta\psi''(q_1 + \tilde{q}_2 - \beta\theta) \left(1 + \frac{\partial \tilde{q}_2}{\partial q_1}\right) \frac{1 - F(\theta)}{f(\theta)}, \end{aligned} \quad (15)$$

$$(1+b)G(r_1^b \setminus q_1(\theta)) + [1 - G(r_1^b \setminus q_1(\theta))] - r^e = 0. \quad (16)$$

The left-hand side of the first equation captures the marginal national gains of increased domestic loan quality, just as in the cooperative case represented by (10). The right-hand side of the equation captures the marginal costs, consisting of marginal resource costs devoted to screening and auditing (the first term) and increased rents (the second term). Compared to the cooperative case, the only difference is that the term accounting for increased rents now contains an additional factor, namely the bank's foreign quality

response $\frac{\partial \tilde{q}_2}{\partial q_1}$. Intuitively, when the national regulator induces the bank to increase the domestic quality level by one unit, the bank adjusts the foreign quality such that the extra resources required to achieve the new domestic level is $1 + \frac{\partial \tilde{q}_2}{\partial q_1}$. From (14) we then see that the increase in rents will be $\beta\psi'' \cdot (1 + \frac{\partial \tilde{q}_2}{\partial q_1})$, and this explains the last term in (15).

The quality levels are substitutes for the bank, and the foreign quality response will then be negative; $\frac{\partial \tilde{q}_2}{\partial q_1} < 0$, see below. By inducing the MNB to increase domestic loan quality, the local regulator provokes a "soft" response by the foreign regulator; the MNB's subsidiary in country 2 is induced to lower its loan quality. This implies that the national regulator perceives the costs associated with increased rents to be smaller than does the supra-national regulator, and hence that he has less of an incentive to distort quality downwards to extract rents. Other things equal, the national regulator will therefore implement a higher domestic quality level for the bank's loans.⁵

Equation (16) is the national regulator's optimality condition for the domestic subsidiary's reserve requirements (R_1). Variations in these requirements do not generate repercussions for the bank's foreign operations, and conditional on the domestic level of loan quality, reserve requirements will therefore be efficient. However, since domestic loan quality will deviate from the level that is optimal under cooperative regulation, reserve requirements will also deviate from the cooperative levels.

In *equilibrium* we must have $\tilde{q}_2(q_1(\theta), \theta) = q_2(\theta)$, and from (13) we then see that the quality response in equilibrium is given by

$$\frac{\partial \tilde{q}_2}{\partial q_1}(q_1(\theta), \theta) = \frac{q'_2(\theta)}{q'_1(\theta) - \beta}$$

where primes denote derivatives. (Writing (13) as $H(q_1, \tilde{q}_2, \theta) = 0$ we have $H_1 + H_2 \frac{\partial \tilde{q}_2}{\partial q_1} = 0$ and $H_1 q'_1 + H_2 q'_2 + H_\theta = 0$, where subscripts on H denote partials. Elimination of H_2 yields the stated formula.) Similar considerations apply for the regulator in country 2, and it then follows that in equilibrium the following conditions hold

$$\begin{aligned} & \int_{r_i^b}^{\bar{r}} [r_i + R_i - 1] dG_{q_i}(r_i \setminus q_i) + (1 + b) \int_0^{r_i^b} [r_i + R_i - 1] dG_{q_i}(r_i \setminus q_i) \\ &= \psi'(\Sigma q_i - \beta\theta) + \beta\psi''(\Sigma q_i - \beta\theta) \left[1 + \frac{q'_j(\theta)}{q'_i(\theta) - \beta} \right] \frac{1 - F(\theta)}{f(\theta)}, \end{aligned} \quad (17)$$

⁵This result follows from our assumption about the cost of improving loan quality. An increase in effort in subsidiary 1 increases the marginal cost of effort in subsidiary 2. Hence, there is an underlying assumption about scarce managerial resources in the MNB. If we instead allowed for "learning-by-doing" effects, the marginal costs of effort in subsidiary 2 would have decreased (if effort in subsidiary 1 increases). In this case, the foreign quality response will be positive, $\frac{\partial \tilde{q}_2}{\partial q_1} > 0$ (complements), and the common-agency effect would cause a deterioration of loan quality.

$$(1 + b)G(r_i^b \backslash q_i(\theta)) + [1 - G(r_i^b \backslash q_i(\theta))] - r^e = 0, \quad (18)$$

where $i, j = 1, 2$, $i \neq j$. Following a procedure similar to Martimort (1992, 1996), one can see that if this system of differential equations defines a pair of nondecreasing loan quality schedules $q_1(\theta)$ and $q_2(\theta)$, and those schedules in addition satisfy a set of implementability conditions, they constitute a pure-strategy differential Nash-equilibrium outcome for the common agency game. The implementability conditions imply that the response effects $\frac{\partial \bar{q}_j}{\partial q_i}$ are negative, which in turn implies, as we have seen, that quality levels are higher than under cooperative regulation.

As commented above, the optimal level of reserve requirements as a function of domestic loan quality is unchanged, and is given by (18). This gives the following result:

Proposition 2 *Compared with international coordination, strategic regulation of a MNB entails (i) higher loan quality and (ii) reduced capital ratios in both countries. The combined effect of these strategic adjustments makes the probability of a banking failure remain the same.*

The reason why regulatory policy induces the MNB to reduce loan quality in the first place, is the rent extraction effect. The regulatory authority, which is concerned about the social cost of its deposit insurance scheme, dislikes leaving extra-normal rents to the MNB since this could instead have been added to the deposit insurance premium. Hence, a more demanding regulatory regime, in the sense of increased incentives for improving loan quality, is desirable from the MNB's point of view. In other words, the MNB benefits from the lack of an internationally coordinated policy towards regulation and supervision of banks.

5 Strategic regulation with ownership effects

So far we have assumed that the regulatory policy towards the MNB is derived from the regulator's concern about the cost of the deposit insurance fund. As noted above, an alternative would be to allow the regulator to care also about the banking profit falling to domestic owners. In that case the objective function of the regulator is given by $W_i = (1 + \lambda)S_i + \delta_i \pi$, where δ_i is the ownership share of country i shareholders, λ is the general equilibrium shadow cost of public funds (assumed equal in the two countries), and S_i is the social cost of the deposit insurance fund (previously denoted W_i).

Following the above procedure, the modified regulatory policy under

coordination will now be given by

$$\begin{aligned} & \left[\int_{r_i^b}^{\bar{r}} [r_i + R_i - 1] dG_{q_i}(r_i \setminus q_i) + (1+b) \int_0^{r_i^b} [r_i + R_i - 1] dG_{q_i}(r_i \setminus q_i) \right] \\ = & \psi' \left(\sum_{i=1}^2 q_i - \beta\theta \right) + \frac{1 + \lambda - \delta_1 - \delta_2}{1 + \lambda} \beta \psi'' \left(\sum_{i=1}^2 q_i - \beta\theta \right) \frac{1 - F(\theta)}{f(\theta)} \end{aligned}$$

We see that the previous analysis captures the case in which the entire MNB is owned by a third country ($\delta_1 = \delta_2 = 0$). As we should expect, a regulator caring for banking profit (in addition to the social cost of running the deposit insurance fund), will be less eager to extract rents, and, hence, loan quality will be higher (everything else equal).

Assuming, instead, strategic regulation by each country, the non-cooperative equilibrium is now characterized by

$$\begin{aligned} & \int_{r_i^b}^{\bar{r}} [r_i + R_i - 1] dG_{q_i}(r_i \setminus q_i) + (1+b) \int_0^{r_i^b} [r_i + R_i - 1] dG_{q_i}(r_i \setminus q_i) \\ = & \psi' + \frac{1 + \lambda - \delta_i}{1 + \lambda} \beta \psi'' \left[1 + \frac{q'_j(\theta)}{q'_i(\theta) - \beta} \right] \frac{1 - F(\theta)}{f(\theta)}, \quad i = 1, 2 \end{aligned}$$

This reveals the ownership effect of strategic banking regulation. As before, we identify the strategic effect due to the soft response of the other country ($\frac{dq_2}{dq_1} < 0$), which explains the increase in loan quality from lack of coordination. However, when bank profits enter the objective function, the importance of rent extraction (induced by lowering loan quality) differs between a domestic regulator and an international regulatory body. A domestic regulator will be more tempted to extract rent since a smaller share of the banking profit enters domestic welfare. This is seen in the above expressions by noticing that $1 + \lambda - \delta_i > 1 + \lambda - \delta_1 - \delta_2$. Hence, the domestic regulator puts more weight on the rent extraction effect than an international regulatory body. This works against the strategic effect, and we cannot generally determine whether loan quality is higher or lower under strategic banking regulation than under international coordination.⁶

From the equilibrium conditions it is clear that the pattern of ownership will have implications for regulatory policy. Following Olsen and Osmundsen (2001), we can derive some comparative results regarding coordinated versus strategic regulation. As already noted, if $\delta_1 = \delta_2 = 0$, ownership effects are absent, and strategic regulation leads to higher loan quality in both countries. Assuming that the solutions vary continuously with the parameters (which is shown to hold in our parametric specification), this will also be the case for sufficiently small values of δ_1 and δ_2 . Assuming instead

⁶A similar effect is present in Martimort (1996b), who studies the implications of a pro-firm bias on the part of regulators in a setting with contract complements.

that $\delta_1 + \delta_2 = 1$, and that $\lambda = 0$, there is no rent extraction under coordination (since profit is equally valuable as surplus in the deposit insurance fund). Under strategic regulation, however, the loan quality is distorted downwards in order to extract rent. The reason is that 1 \$ increase in the deposit insurance premium, reduces domestic profit by δ_i \$ (<1). Hence, lack of international coordination banking regulation will causes a downward distortion in loan quality. Again, by assuming continuity, that same will hold for λ sufficiently small and $\delta_1 + \delta_2$ sufficiently close to 1. This can be summarized as follows:

Proposition 3 *Suppose two countries are symmetric. (i) If both λ and the outside (third country) ownership share $1 - \delta_1 - \delta_2$ are sufficiently small, then strategic banking regulation leads to lower loan quality (and higher capital ratios) in both countries compared to a situation with international coordination. (ii) If the outside ownership share is sufficiently large, then strategic banking regulation leads to higher loan quality (and lower capital ratios) in both countries compared to a situation with international coordination.*

The effect of ownership structure on banking regulation may have consequences for what kind of ownership a MNB may end up with. As seen from (19), if the shareholders in both countries symmetrically sell their shares to third-country shareholders, then the MNB will be induced to lower its loan quality (since rent extraction becomes more important for the national regulators), and banking profit ($\pi(\theta)$) will tend to fall. A MNB, therefore, will benefit from being owned by shareholders from the markets in which the MNB operates. If a third-country shareholder sets up a new foreign bank with subsidiaries in each of these foreign markets, the charter-value of the bank will actually increase if it is sold to shareholders from these countries.

6 A parametric specification

In this section we derive explicit solutions of regulatory policies by assuming specific functional forms. We suppose here that $G(r \setminus q_i) = G(\frac{r}{Q(q_i)})$ where $G(t)$ is a CDF on some interval $[0, \bar{t}]$ and $Q(q_i)$ is increasing. Then a higher quality level q_i will shift the distribution of returns r_i to a more favorable one in terms of first-order stochastic dominance.

6.1 Coordinated regulation

In the appendix we show that the optimality condition for q_i can now be written as

$$K(r^e, b)Q'(q_i) = \psi'(q_1 + q_{2i} - \beta\theta) + \beta\psi''(q_1 + q_2 - \beta\theta)\frac{1 - F(\theta)}{f(\theta)}, \quad i = 1, 2$$

where $K(r^e, b)$ is increasing in r^e and decreasing in b .

Further, we assume that $\psi()$ is quadratic; $\psi(e) = \frac{c}{2}e^2$, and that θ is uniform on $[0, 1]$. Suppose moreover that $Q'(q_i) = Q_1 q_i + Q_2$, $Q_i \geq 0$. The Q_k -parameters can be seen as measures of the marginal productivity of quality with respect to improving loan quality. Then optimal q_i under coordinated regulation is given by

$$K(r^e, b)(Q_1 q_i + Q_2) = c(q_1 + q_2 - \beta\theta) + \beta c(1 - \theta), \quad i = 1, 2$$

For symmetric countries (where $q_1 = q_2$) there is a well defined solution (denoted by q_C) provided $K(r^e, b)Q_1 < 2c$, and then

$$q_C(\theta) = \frac{KQ_2 + c\beta(2\theta - 1)}{2c - KQ_1}, \quad K = K(r^e, b), \quad KQ_1 < 2c$$

Comparing with the first-best solution (with symmetric information about θ)

$$q_{FB}(\theta) = \frac{KQ_2 + c\beta\theta}{2c - KQ_1}$$

we see that the quality levels are distorted downwards under asymmetric information; $q_C(\theta) < q_{FB}(\theta)$. The relative distortion $\frac{q_{FB}(\theta) - q_C(\theta)}{q_{FB}(\theta)} = \frac{c\beta(1-\theta)}{KQ_2 + c\beta\theta}$ increases with $c\beta$ and decreases with KQ_2 .

6.2 Non-cooperative regulation

The equilibrium condition for q_1 is in this case (this follows from (19)):

$$K(r^e, b)(Q_1 q_i + Q_2) = c(q_1 + q_2 - \beta\theta) + \beta c \left[1 + \frac{q'_2(\theta)}{q'_1(\theta) - \beta} \right] (1 - \theta). \quad (20)$$

Similarly for q_2 . We seek linear (and symmetric) solutions for the quality profiles

$$q_1(\theta) = q_2(\theta) = q_{nc}(\theta) = \bar{q} - (1 - \theta)q' \quad (21)$$

As shown in the appendix this yields

$$\bar{q} = \frac{KQ_2 + c\beta}{2c - KQ_1} = q_{FB}(\bar{\theta})$$

while the solution for q' that satisfies the implementability condition $2q' \leq \beta$ is

$$q' = \frac{\beta}{2(2c + K_1)} \left(K_1 + 5c - \sqrt{(K_1^2 + 2K_1c + 9c^2)} \right), \quad K_1 = K(r^e, b)Q_1$$

The slope q' of the equilibrium quality profile $q_{nc}(\theta)$ is increasing in β and decreasing in K_1 . Comparing with the cooperative solution $q_C(\theta)$, we find

that the latter profile is steeper ($q' < q'_C$), and, hence, that there is more provision of quality in the non-cooperative case ($q_{nc}(\theta) > q_C(\theta)$). This is due to the presence of a strategic effect in the latter case.

We observe that, for $Q_1 = 0$ we have $q' = \frac{\beta}{2}$ and hence $q_{nc}(\theta) = q_{FB}(\theta)$. In this (extreme) case the strategic effect (the foreign quality adjustment) is so strong that none of the national regulators finds it worth while to distort domestic quality from the first-best level. Any domestic distortion would be completely offset by the bank switching more of its quality enhancing resources to the subsidiary in the other country. In this case the quality variables are perfect substitutes for the bank, and it isn't possible for any of the non-coordinated regulators to use unilateral quality distortions to extract rents from the MNB. In equilibrium there will thus be no distortions, and the first-best outcome is realized. With coordinated efforts the two regulators will however be able to extract rents this way, and in fact the optimal relative distortion $\frac{q_{FB}(\theta) - q_C(\theta)}{q_{FB}(\theta)}$ was seen to be independent of the parameter Q_1 .

For positive values of Q_1 , i.e. when the marginal productivity of quality is increasing with more quality in each country, the quality variables are no longer perfect substitutes for the bank, and then it becomes possible for each regulator to extract rents by unilateral distortions of domestic quality. The non-cooperative equilibrium will then involve distortions from the first-best in each country, but the distortions will be smaller than in the cooperative case.

7 Conclusion

In this paper we have analyzed how entry of multi-national bank (MNB) subsidiaries affects banking supervision and regulation. When a MNB expands internationally with subsidiaries, the MNB operates under the legislation of several countries - both the home country and the host countries. Although these countries have agreed upon minimum standards and supervisory principles, such as in the EU directives or the Basle Accords, substantial degrees of freedom are still left to national regulators. For instance, figures presented in BIS (1999) shows that there is no clear evidence that the variation in capital ratios between G-10 banks has been reduced since the 1988 Basel Accord.

Host country regulation of MNBs subsidiaries is shown to create cross-border externalities, where the supervisors and regulators in one country will be concerned with the standards in the home country and in other host countries. Our main results are as follows.

First, there is a regulatory induced contagion of macroeconomic shocks between the two countries. If one of the two countries experiences an economic downturn, the regulatory induced quality of the loan portfolios in

both countries are affected with equal strength. Banks of sufficiently high (low) intrinsic quality will experience a deterioration (improvement) of induced quality of the loan portfolio in *both markets* if one of the markets experiences an economic downturn.

Second, lack of international coordination of banking regulation works to lower capital adequacy requirements. However, in equilibrium regulators respond by increasing the incentives to improve asset quality, making the probability of banking failure insensitive to the decentralized nature of banking regulation.

Third, ownership of the MNB is shown to be of importance for the outcome of strategic banking regulation. If the MNB is owned by shareholders from outside the market operated by the bank ("third-country shareholders"), the regulatory regime becomes more distortive since regulators become more eager to extract banking profits. Consequently, with more "inside-shareholders", the regulatory policy becomes more pro-bank industry inclined.

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Appendix

Here we derive the claims made in section 6.

Note first that (since $r_i^b = 1 - R_i$) conditions (10),(11) can be written (using integration by parts)

$$\begin{aligned} & \frac{\partial}{\partial q_i} E r_i + b \frac{\partial}{\partial q_i} \left(- \int_0^{r_i^b} G(r_i \setminus q_i) dr_i \right) \\ = & \psi' \left(\sum_{i=1}^2 q_i - \beta \theta \right) + \beta \psi'' \left(\sum_{i=1}^2 q_i - \beta \theta \right) \frac{1 - F(\theta)}{f(\theta)}, \quad i = 1, 2 \end{aligned} \quad (22)$$

$$bG(r_i^b \setminus q_i(\theta)) + 1 - r^e = 0, \quad i = 1, 2 \quad (23)$$

Given that $G(r \setminus q_i) = G(\frac{r}{Q(q_i)})$ where $G(t)$ is a CDF on some interval $[0, \bar{t}]$ and $Q(q_i)$ is increasing, then $r = tQ$ and so $E(r \setminus q_i) = Q(q_i)\gamma$, where $\gamma = Et$ a constant (independent of q). Moreover we have

$$\begin{aligned} -\frac{\partial}{\partial q_i} \int_0^{r_i^b} G\left(\frac{r}{Q(q_i)}\right) dr &= \int_0^{r_i^b} G'\left(\frac{r}{Q}\right) \frac{r}{Q^2} dr Q'(q_i) \\ &= Q'(q_i) \int_0^{r_i^b/Q} t G'(t) dt = Q'(q_i) k(r^e, b) \end{aligned}$$

where $k(r^e, b)$ is a constant independent of q_i ; this follows from (23), which for the present specification says that $bG(\frac{r_i^b}{Q}) = r^e - 1$. We see that $k(r^e, b)$ is an increasing function of $\frac{r^e - 1}{b}$. Letting $K(r^e, b) = \gamma + k(r^e, b)$ the optimality condition for q_i in the cooperative case can then be written as stated in the text, and $K(r^e, b)$ is also an increasing function of $\frac{r^e - 1}{b}$.

For the non-cooperative case, substituting (21) into (20) yields

$$KQ_1 (\bar{q} - (1 - \theta)q') + KQ_2 = 2c (\bar{q} - (1 - \theta)q') - c\beta\theta + \beta c \left[1 + \frac{q'}{q' - \beta} \right] (1 - \theta).$$

Collecting terms we get two equations for \bar{q} and q' :

$$\begin{aligned} KQ_1 \bar{q} + KQ_2 &= 2c\bar{q} - c\beta \\ KQ_1 q' &= -2cq' + c\beta + \beta c \left[1 + \frac{q'}{q' - \beta} \right] \end{aligned}$$

This yields the solutions stated in the text.